Chapter 4
Uncertainty of Discharge-Probability Function

4-1. Function Development

a. A discharge or stage-probability function is critical to evaluation of flood damage reduction plans. The median function is used for the analytical method. The manner in which the function is defined depends on the nature of the available data. A direct analytical approach is used when a sample (such as stream gauge records of maximum annual discharges) is available and it fits a known statistical distribution such as log Pearson III. Other approaches are required if recorded data are not available or if the recorded data do not fit a known distribution. These approaches include using the analytical method after defining parameters of an adopted discharge-probability function generated by various means and the graphical or "eye fit" approach for fitting the function through plotting position points. The synthetic statistics approach is applied when the statistics for an adopted discharge-probability function are consistent with hydrologically and meteorologically similar basins in the region. The adopted function may be determined using one or more of the methods presented in Table 4-1. The graphical approach is commonly used for regulated and stage-probability functions whether or not they are based on stream gauge records or computed and stage-probability functions whether or not they are based on stream gauge records or computed from simulation analysis.

b. The without-project conditions discharge-probability functions for the base years are derived initially for most studies and become the basis of the analysis for alternative plans and future years. These functions may be the same as the without-project base year conditions or altered by flood damage reduction measures and future development assumptions. The uncertainty associated with these functions may be significantly different, in most instances greater.

c. Flood damage reduction measures that directly affect the discharge or stage-probability function include reservoirs, detention storage, and diversions. Other measures, if implemented on a large scale, may also affect the functions. Examples are channels (enhanced conveyance), levees (reduction in natural storage and enhanced conveyance), and relocation (enhanced conveyance).

4-2. Direct Analytical Approach

a. General. The direct analytical approach is used when a sample of stream gauge annual peak discharge values are available and the data can be fit with a statistical distribution. The median function is used in the risk-based analysis. The derived function may then be used to predict specified exceedance probabilities. The approach used for Corps studies follows the U.S. Water Resources Council's recommendations for Federal planning involving water resources presented in publication Bulletin 17B (Interagency Advisory Committee on Water Data 1982) and in EM 1110-2-1415 and ER 1110-2-1450.

Table 4-1
Procedures for Estimating Discharge-Probability Function Without Recorded Events (adapted from USWRC (1981))

Method	Summary of Procedure
Transfer	Discharge-probability function is derived from discharge sample at nearby stream. Each quantile (discharge value for specified probability) is extrapolated or interpolated for the location of interest.
Regional estimation of individual quantiles or of function parameters	Discharge-probability functions are derived from discharge samples at nearby gauged locations. Then the function parameters or individual quantiles are related to measurable catchment, channel, or climatic characteristics via regression analysis. The resulting predictive equations are used to estimate function parameters or quantiles for the location of interest.
Empirical equations	Quantile (flow or stage) is computed from precipitation with a simple empirical equation. Typically, the probability of discharge and precipitation are assumed equal.
Hypothetical frequency events	Unique discharge hydrographs due to storms of specified probabilities and temporal and areal distributions are computed with a rainfall-runoff model. Results are calibrated to observed events or discharge-probability relations at gauged locations so that probability of peak hydrograph equals storm probability.
Continuous simulation	Continuous record of discharge is computed from continuous record of precipitation with rainfall-runoff model, and annual discharge peaks are identified. The function is fitted to series of annual hydrograph peaks, using statistical analysis procedures.

b. Uncertainty of distribution parameters due to sampling error.

(1) Parameter uncertainty can be described probabilistically. Uncertainty in the predictions is attributed to lack of perfect knowledge regarding the distribution and parameters of the distribution. For example, the log Pearson type III distribution has three parameters: a location, a scale, and a shape parameter. According to the Bulletin 17B guidance, these are estimated with statistical moments (mean, standard deviation, and coefficient of skewness) of a sample. The assumption of this so-called method-of-moments parameter-estimation procedure is that the sample moments are good estimates of the moments of the population of all possible annual maximum discharge values. As time passes, new observations will be added to the sample, and with these new observations the estimates of the moments, and hence the distribution parameters, will change. But by analyzing statistically the sample moments, it is possible to draw conclusions regarding the likelihood of the true magnitude of the population moments. For example, the analysis might permit one to conclude that the probability is 0.90 that the parent population mean is between 10,000 m^3/s and 20,000 m^3/s. As the discharge-probability function parameters are a mathematical function of the moments, one can then draw conclusions about the parameters through mathematical manipulation. For example, one might conclude that the probability is 0.90 that the location parameter of the log Pearson type III model is between a specified lower limit and a specified upper limit. Carrying this one step further to include all three parameters permits development of a description of uncertainty in the frequency function itself. And from this, one might conclude that the probability is 0.90 that the 0.01-probability discharge is between 5,000 m^3/s and 5,600 m^3/s. With such a description, the sampling described in Chapter 2 can be conducted to describe the uncertainty in estimates of expected annual damage and annual exceedance probability.

(2) Appendix 9 of Bulletin 17B presents a procedure for approximately describing, with a statistical distribution, the uncertainty with a log-Pearson type III distribution with parameters estimated according to the Bulletin 17B guidelines. This procedure is summarized in Table 4-2; an example application is included in Tables 4-3 and 4-4.

(3) The sampling methods described in Chapter 2 require a complete description of error or uncertainty about the median frequency function. To develop such a description, the procedure shown in Table 4-2 can be repeated for various values of *C*, the confidence level. Table 4-3, for example, is a tabulation of the statistical model that describes uncertainty of the 0.01-probability quantile for Chester Creek, PA.

c. Display of uncertainty. The probabilistic description of discharge-probability function uncertainty can be displayed with confidence limits on a plotted function, as shown in Figure 4-1. These limits are curves that interconnect discharge or stage values computed for each exceedance probability using the procedure shown in Table 4-2, with specified values of *C* in the equations. For example, to define a so-called *95-percent-confidence limit*, the equations in Table 4-2 are solved for values of *P* with *C* constant and equal to 0.95. The resulting discharge values are plotted and interconnected. Although such a plot is not required for the computations proposed herein, it does illustrate the uncertainty in estimates of quantiles.

4-3. Analytical Approach

The analytical approach for adopted discharge-probability functions, also referred to as the synthetic approach, is described in Bulletin 17B (Interagency Advisory Committee 1982). It is used for ungauged basins when the function is derived using the transfer, regression, empirical equations, and modeling simulation approaches presented in Table 4-1 and when it is not influenced by regulation, development, or other factors. The discharge-probability function used is the median function and is assumed to fit a log Pearson Type II distribution by deriving the mean, standard deviation, and generalized skew from the adopted function defined by the estimated 0.50-, 0.10-, and 0.01-exceedance probability events. Assurance that the adopted function is valid and is properly fitted by the statistics is required. If not, the graphical approach presented in the next section should be applied. The value of the function is expressed as the equivalent record length which may be equal to or less than the record of stream gauges used in the deviation of the function. Table 4-5 provides guidance for estimating equivalent record lengths. The estimated statistics and equivalent record length are used to calculate the confidence limits for the uncertainty analysis in a manner previously described under the analytical approach.

4-4. Graphical Functions

a. Overview. A graphical approach is used when the sample of stream gauge records is small, incomplete,

Table 4-2
Procedure for Confidence Limit Definition (from Appendix 9, Bulletin 17B)

The general form of the confidence limits is specified as:

$$U_{P,C}\ (X) = \overline{X} + S\ (K^{U}_{P,C})$$

$$L_{P,C}\ (X) = \overline{X} + S\ (K^{L}_{P,C})$$

in which X and S and are the logarithmic mean and standard deviation of the final estimated log Pearson Type III discharge-probability function, and $K^{U}{}_{P,C}$ and $K^{L}{}_{P,C}$ are upper and lower confidence coefficients. [Note: P is the exceedance probability of X, and C is the probability that $U_{P,C} > X$ and that $L_{P,C} < X$.]

"The confidence coefficients approximate the non-central t-distribution. The non-central-t variate can be obtained in tables (41,42), although the process is cumbersome when G_W is non-zero. More convenient is the use of the following approximate formulas (32, pp. 2-15) based on a large sample approximation to the non-central t-distribution (42).

$$K^{U}_{P,C} = \frac{K_{G_w,\ P} + \sqrt{K^2 G_w\ ,\ P - ab}}{a}$$

$$K^{L}_{P,C} = \frac{K_{G_w,\ P} + \sqrt{K^2 G_w\ ,\ P - ab}}{a}$$

in which:

$$a = 1 - \frac{Z_C^2}{2\ (N - 1)}$$

$$b = K^2_{G_{W},\ P} - \frac{Z_C^2}{N}$$

and Z_C is the standard normal deviate (zero-skew Pearson Type III deviate with cumulative probability, C (exceedance probability 1-C). The systematic record length N is deemed to control the statistical reliability of the estimated function and is to be used for calculating confidence limits even when historic information has been used to estimate the discharge-probability function.

Examples are regulated flows, mixed populations such as generalized rainfall and hurricane events, partial duration data, development impacts, and stage exceedance probability. The graphical method does not yield an analytical representation of the function, so the procedures described in Bulletin 17B cannot be applied to describe the uncertainty. The graphical approach uses plotting positions to define the relationship with the actual function fitted by "eye" through the plotting position points. The uncertainty relationships are derived using an approach referred to as order statistics (Morgan and Henrion 1990). The uncertainty probability function distributions are assumed normal, thus requiring the use of the Wiebull's plotting positions, representing the expected value definition of the function, in this instance.

b. Description with order statistics. The order statistics method is used for describing the uncertainty for frequency functions derived for the graphical approach. The method is limited to describing uncertainty in the estimated function for the range of any observed data, or if none were used, to a period of record that is equivalent in information content to the simulation method used to derive the frequency function. Beyond this period of record, the method extrapolates the uncertainty description using asymptotic approximations of error distributions. The procedure also uses the equivalent record length concepts described in Section 4-3 and presented in Table 4-5.

Table 4-3
Example of Confidence Limit Computation (from Appendix 9, Bulletin 17B)

The 0.01 exceedance probability discharge for Chester Creek at Dutton Mill gauge is 18,990 cfs. The discharge-probability curve there is based on a 65-year record length (N = 65), with mean of logs of annual peaks (X) equal to 3.507, standard deviation of logs (S) equal to 0.295, and adopted skew (G_w) equal to 0.4. Compute the 95-percent confidence limits for the 0.01 exceedance probability event.

Procedure: From a table of standard normal deviates, Z_c for the 95-percent confidence limit (C = 0.95) is found to be 1.645. For the 0.01 probability event with G_w = 0.4, the Pearson deviate, $K_{Gw,P} = K_{0.4,0.01}$ is found to be 2.6154. Thus a and b are computed as

$$a = 1 - \frac{(1.645)^2}{2\ (65 - 1)} = 0.9789$$

$$b = (2.6154)^2 - \frac{(1.645)^2}{65} = 6.7987$$

The Pearson deviate of the upper confidence limit for the 0.01-probability event is

$$K^{U}_{0.01,0.95} = \frac{2.6154 + \sqrt{(2.164)^2 - (6.7987)\ (0.9789)}}{0.9789} = 3.1112$$

and the Pearson deviate of the lower confidence limit for the 0.01-probability event is

$$K^{L}_{0.01,0.95} = \frac{2.6154 - \sqrt{(2.164)^2 - (6.7987)\ (0.9789)}}{0.9789} = 2.2323$$

Thus the upper confidence-limit quantile is

$$U_{0.01,0.95}\ (X) = 3.507 + 0.295\ (3.1112) = 4.4248$$

and the lower quantile is

$$L_{0.01,0.95}\ (X) = 3.507 + 0.295\ (2.2323) = 4.1655$$

The corresponding quantiles in natural units are 26,600 cfs and 14,650 cfs, respectively.

Table 4-4
Distribution of Estimates of Chester Creek 0.01-Probability Quantile

Exceedance Probability	Discharge, cms
0.9999	320
0.9900	382
0.9500	415
0.9000	437
0.7000	491
0.5000	538
0.3000	592
0.1000	694
0.0500	753
0.0100	895
0.0001	1,390

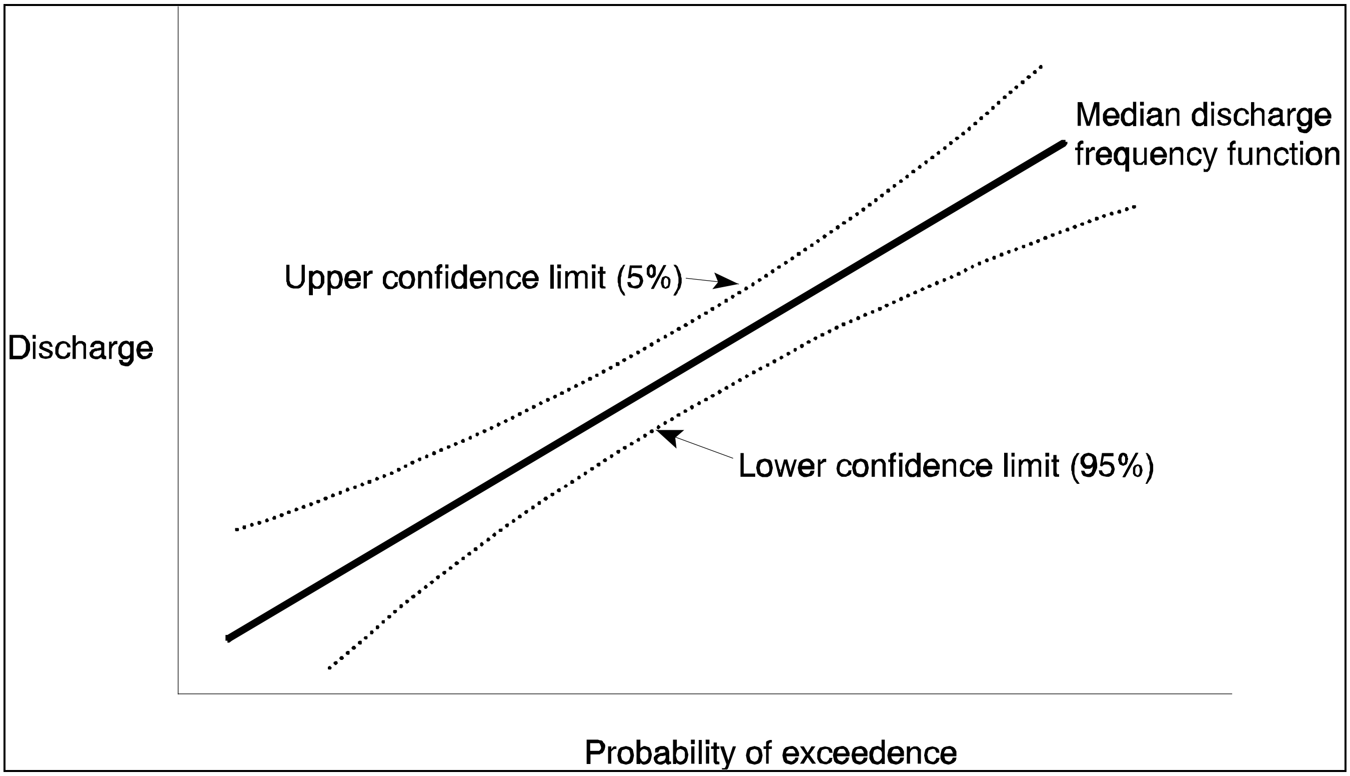

Figure 4-1. Confidence limits

Table 4-5
Equivalent Record Length Guidelines

Method of Frequency Function Estimation	Equivalent Record Length[1]
Analytical distribution fitted with long-period gauged record available at site	Systematic record length
Estimated from analytical distribution fitted for long-period gauge on the same stream, with upstream drainage area within 20% of that of point of interest	90% to 100% of record length of gauged location
Estimated from analytical distribution fitted for long-period gauge within same watershed	50% to 90% of record length
Estimated with regional discharge-probability function parameters	Average length of record used in regional study
Estimated with rainfall-runoff-routing model calibrated to several events recorded at short-interval event gauge in watershed	20 to 30 years
Estimated with rainfall-runoff-routing model with regional model parameters (no rainfall-runoff-routing model calibration)	10 to 30 years
Estimated with rainfall-runoff-routing model with handbook or textbook model parameters	10 to 15 years

[1] Based on judgment to account for the quality of any data used in the analysis, for the degree of confidence in models, and for previous experience with similar studies.